

Detection of massive multi-particle beams by two-particle ionization

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Abstract

Multi-photon absorption is a well-known phenomenon. With atom lasers a similar process could take place for massive particles, the ionization of an atom or molecule by the successive interaction with various particles. This process would lead to multi-particle detection events for incident multi-particle beams. We show that two-particle detections would introduce a correction (proportional to the fourth power of the wave-function modulus) to the usual one-particle detection probability (only proportional to the second power).

Keywords: Detection of multi-particle beams; Ionization by massive particles; Atom lasers

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1 Introduction

The advent of the laser, providing a source of light beams with high degrees of coherence was a major advance in the understanding and manipulation of the quantum nature of light. Similarly, the recent arrival of atom lasers could open new conceptual and experimental areas in the field of atom optics, the study of quantum properties of matter waves [1]. The atom lasers provide sources of matter waves with many particles in the same state. It emerges naturally the question if the usual theory of detection will suffice to correctly describe measurements on multi-particle beams or if new effects could be present. We analyze here a new aspect of the problem, the possible existence of two-particle detection processes (single detection events produced by the interaction with two particles of the beam). At this point we return to quantum optics.

One of the advances associated with the laser was the improvement in the study of multi-photon absorption, a central issue in non-linear optics. An atom can absorb in consecutive steps various incident photons [2]. The beams used in two-photon absorption can be obtained from different classical light sources. However, the intensity is in general weak and the non-linear terms can be neglected. Only, for light from a laser source the non-linear terms are in general relevant. Having at our disposal matter wave beams with various particles in the same state, we can pose the question if similar properties could be observed for particles obeying Schrödinger's equation. Massive particles can ionize atoms and molecules. One expects that when these atoms and molecules interact with multi-particle beams there can be processes where they are not ionized by a single particle, but by successive interactions with the particles of the beam. This property could have implications in the context of detection theory. As many detectors of massive particles are based on the ionization induced by the probe particles these processes would lead to *multi-detection events* (single detection events, for instance single detection clicks or spots, produced by various particles).

We shall show as the main result of the paper that the detection probabilities expected in the presence of *multi-detection events* are, in principle, distinguishable from those associated with *single-detection events*, opening the door to a possible experimental verification of these phenomena. To be concrete, the first type of event is proportional for particles in the same state to the fourth power of the modulus of the wavefunction, whereas for the second type it is only proportional to the second power. This behaviour is similar to that found in quantum optics, where the probabilities of single- and double-absorption are proportional to the intensity and to the squared intensity. In particular, the interference patterns for one- or two-particle incident beams would be slightly different. The results obtained here for detection by ionization would remain unchanged for any type of detection scheme with *multi-detection events*.

2 Multi-detection events

Several common methods of detection for massive particles rest on the ionizations these particles can induce on the atoms or molecules composing the detector. Well-known examples are those of plates (used for both massless and massive particles), Geiger counters or bubblechambers.

When the underlying physical mechanism in the detection process is the ionization we can think along very similar lines to the multi-photon absorption. In the case of a single incident particle the interaction with the detector produces directly the ionization (or fails to produce ionization). With two incident particles, we can have other ways to the final ionization. The first incident particle can carry out the atoms or molecules of the detector to an excited state, but without ionization. Later, the interaction with a second particle definitively

ionizes the detector. The similarity with the multiple absorption of photons is clear.

Usually atom laser beams and in general multi-particle beams are obtained from Bose-Einstein condensates [3]. The components of these condensates are neutral, whereas the effects of ionization are most times associated with charged particles. To convert the original beam into one with charged particles we can use a laser which ionizes the components of the beam.

From now on we concentrate on the possible modifications of the detection probabilities associated with *multi-detection events*.

3 Detection probabilities

Let us consider a small size detector placed at a given position, which is used to measure the probability of finding at that position particles [4]. The incident beam is in a multi-particle state.

The probabilities for single and double detection events at point \mathbf{r} (the position of the detector) and time t , in spin states μ or η , are given by the well-known expressions:

$$P_{sin}(\mathbf{r}, t; \mu \vee \eta) = \sum_{\xi=\mu, \eta} \frac{\langle I | \hat{\psi}_{\xi}^+(\mathbf{r}, t) \hat{\psi}_{\xi}(\mathbf{r}, t) | I \rangle}{\langle I | I \rangle} \quad (1)$$

and

$$P_{dou}(\mathbf{r}, t; \mu, \eta) = \frac{\langle I | \hat{\psi}_{\mu}^+(\mathbf{r}, t) \hat{\psi}_{\eta}^+(\mathbf{r}, t) \hat{\psi}_{\eta}(\mathbf{r}, t) \hat{\psi}_{\mu}(\mathbf{r}, t) | I \rangle}{\langle I | I \rangle} \quad (2)$$

with $|I\rangle$ the state of the incident beam (expressed in Fock's space) and where $\mu \vee \eta$ means that we can detect the particle in any of the two spin states.

The scheme based on the two above equations is the general one to evaluate detection probabilities in the second quantization framework, but it is specially well suited to study systems where the detection occurs through the absorption of the particles to be measured. As a matter of fact, the method follows closely the seminal work of Glauber [5] to describe the detection of photons by photoionization. In the usual detection schemes based on ionization (photographic plates, Geiger counters and bubblechambers) the detected particle becomes finally mixed with the rest of components of the detector, i. e., it is absorbed by the detector.

In the above equations $\hat{\psi}_{\mu}(\mathbf{r}, t)$ is the Schrödinger field operator for spin state μ . Assuming that we work in a finite volume space, it is given by the expression:

$$\hat{\psi}_{\mu}(\mathbf{r}, t) = \sum_{\mathbf{n}} \psi_{\mathbf{n}\mu}(\mathbf{r}) \hat{a}_{\mathbf{n}\mu}(t) \quad (3)$$

with $\psi_{\mathbf{n}\mu}(\mathbf{r})$ a complete set of orthonormal stationary wavefunctions characterized by the discrete index \mathbf{n} (the momenta available to the particle) and spin

state μ . If we would work in R^3 instead of the finite volume, the summation should be replaced by an integration and the discrete index by a continuous one. The time dependence of Schrödinger's field is contained in the annihilation operator, which can be expressed as $\hat{a}_{\mathbf{n}\mu}(t) = \hat{a}_{\mathbf{n}\mu} \exp(-iE_{\mathbf{n}\mu}t/\hbar)$ with $\hat{a}_{\mathbf{n}\mu}$ the operator at $t = 0$ and $E_{\mathbf{n}\mu}$ the energy of the stationary state. The (anti)commutation relations are $[\hat{a}_{\mathbf{n}\mu}, \hat{a}_{\mathbf{m}\Omega}^\dagger]_\mp = \delta_{\mathbf{n}\mathbf{m}}^3 \delta_{\mu\Omega}$, with the upper sign of the double expression valid for bosons and the lower one for fermions.

The single and double detection events can be seen as two available channels for the interaction between the incident beam and the detector. The total probability of detection can be expressed as:

$$P_{det}(\mathbf{r}, t; \mu, \eta) = \alpha_{sin} P_{sin}(\mathbf{r}, t; \mu \vee \eta) + \alpha_{dou} P_{dou}(\mathbf{r}, t; \mu, \eta) \quad (4)$$

where α_{sin} and α_{dou} are two phenomenological coefficients characterizing the weight of the two channels. They can be determined experimentally.

Writing the detection probability in this form we implicitly assume that the records for a double detection at \mathbf{r} ($P_{dou}(\mathbf{r})$) and for two detections at close points \mathbf{r} and \mathbf{R} inside the detector ($P_{sin}(\mathbf{r})$ and $P_{sin}(\mathbf{R})$) can be distinguished. For the Geiger counter this condition is fulfilled because the spark currents (the records) are clearly different for both situations. Similarly, in the case of the bubblechamber for two ionizations we would have two lines of bubbles (the records), whereas for a double ionization we would only have one line. The situation is not so favorable for photographic plates, where the typical size of the spots (the records) imposes a limit to the possibility of distinguishing both types of events. However, the typical size of the spots is very small and the error introduced can be neglected.

Using the above expressions we can easily evaluate the detection probabilities for the initial two-particle state

$$|I\rangle = \sum_{\mathbf{n}, \mathbf{m}} b_{\mathbf{n}} d_{\mathbf{m}} \hat{a}_{\mathbf{n}\sigma}^\dagger \hat{a}_{\mathbf{m}\Omega}^\dagger |0\rangle \quad (5)$$

where \mathbf{n} and \mathbf{m} represent the complete set of momenta available to the particles. On the other hand, σ and Ω are the spin states of the two particles. By simplicity, we assume that the coefficients representing the momenta distributions, $b_{\mathbf{n}}$ and $d_{\mathbf{m}}$, are independent of the spin states. Finally, $|0\rangle$ represents the vacuum state.

Using this form of the two-particle incident beam we assume that it is composed of two particles, one in state

$$\psi_{(b;\sigma)}(\mathbf{r}, t) = \sum_{\mathbf{n}} b_{\mathbf{n}} \psi_{\mathbf{n}\sigma}(\mathbf{r}) \exp(-iE_{\mathbf{n}\mu}t/\hbar) \quad (6)$$

with $\psi_{\mathbf{n}\sigma}$ a complete set of orthonormal stationary wavefunctions in spin state σ . The other particle is in the state $\psi_{(d;\Omega)}(\mathbf{r}, t)$, given by a similar expression with obvious modifications.

The denominator of the detection probabilities is

$$\langle I|I \rangle = \pm 1 + \delta_{\sigma\Omega} | \langle d|b \rangle |^2 \quad (7)$$

with $\langle d|b \rangle = \sum_{\mathbf{n}} d_{\mathbf{n}}^* b_{\mathbf{n}}$.

A simple calculation gives:

$$P_{sin}(\mathbf{r}, t; \mu \vee \eta) = \mathcal{P}(\mathbf{r}, t; \mu) + \mathcal{P}(\mathbf{r}, t; \eta) \quad (8)$$

with

$$\mathcal{P}(\mathbf{r}, t; \mu) = \frac{\pm \delta_{\mu\Omega} |\psi_{(d;\Omega)}(\mathbf{r}, t)|^2 \pm \delta_{\mu\sigma} |\psi_{(b;\sigma)}(\mathbf{r}, t)|^2 + 2\delta_{\sigma\Omega} \delta_{\mu\sigma} \text{Re}(\langle d|b \rangle \psi_{(b;\sigma)}^*(\mathbf{r}, t) \psi_{(d;\Omega)}(\mathbf{r}, t))}{\pm 1 + \delta_{\sigma\Omega} | \langle d|b \rangle |^2} \quad (9)$$

and a similar expression for $\mathcal{P}(\mathbf{r}, t; \eta)$ with μ replaced by η . On the other hand, we have

$$P_{dou}(\mathbf{r}, t; \mu, \eta) = \frac{|\psi_{(d;\Omega)}(\mathbf{r}, t)|^2 |\psi_{(b;\sigma)}(\mathbf{r}, t)|^2 (2\delta_{\eta\Omega} \delta_{\mu\sigma} \delta_{\sigma\eta} \delta_{\Omega\mu} \pm \delta_{\sigma\mu} \delta_{\eta\Omega} \pm \delta_{\sigma\eta} \delta_{\mu\Omega})}{\pm 1 + \delta_{\sigma\Omega} | \langle d|b \rangle |^2} \quad (10)$$

The physical meaning of all these expressions is discussed in next section.

4 Discussion

First, we consider the probability of single detections. It must be noted that both the numerator and the denominator of the expression for single detection probabilities are negative for fermions, but the complete expression remains always positive (these properties can easily be verified following a similar procedure to that presented in the Appendix of Ref. [6]).

We consider the case $\mathcal{P}(\mathbf{r}, t; \mu)$ (the discussion of $\mathcal{P}(\mathbf{r}, t; \eta)$ is similar). Three terms contribute to this single detection probability. Those proportional to $|\psi_{(d;\Omega)}|^2$ and $|\psi_{(b;\sigma)}|^2$ represent the probabilities of detecting only one of the particles. On the other hand, the third term has the typical form of the interference between the two previous alternatives for the detection (to detect the particle in state $(d; \Omega)$ or in $(b; \sigma)$). This last term is only present when $\sigma = \Omega = \mu$ and $\langle d|b \rangle \neq 0$. The last condition implies that both particles must have common modes. When there are common modes the detector does not distinguish if these modes correspond to one or the other particle. In the presence of indistinguishable alternatives quantum theory gives rise to interference phenomena (see Ref. [6], where an extensive analysis of this type of interference has been carried out).

Now, we consider the probability of double detections. Again, taking into account that $2\delta_{\eta\Omega} \delta_{\mu\sigma} \delta_{\sigma\eta} \delta_{\Omega\mu} \leq \delta_{\eta\Omega} \delta_{\mu\sigma} + \delta_{\sigma\eta} \delta_{\Omega\mu}$ it is simple to see that the

expression is non-negative for fermions. This term represents a different type of contribution to the detection probability. For instance, in the case $\sigma \neq \Omega$, $\mu = \sigma$, $\eta = \Omega$ and $b = d$ ($b_{\mathbf{n}} = d_{\mathbf{n}}$) the total detection probability (valid for both fermions and bosons) is

$$P_{det}(\mathbf{r}, t; \mu, \eta) = \alpha_{sin}(|\psi_{(b;\mu)}(\mathbf{r}, t)|^2 + |\psi_{(b;\eta)}(\mathbf{r}, t)|^2) + \alpha_{dou}|\psi_{(b;\mu)}(\mathbf{r}, t)|^2|\psi_{(b;\eta)}(\mathbf{r}, t)|^2 \quad (11)$$

The second term in the r. h. s. of this expression contains the product of the squared modulus of the two wavefunctions. This analytical form differs from the usual squared dependence on the modulus of the single term.

Other interesting situation is when the incident beam is composed of two bosons in the same state ($b = d$ and $\mu = \eta = \sigma = \Omega$). This situation corresponds to a two-boson laser and the total probability is given by

$$P_{det}(\mathbf{r}, t; \mu, \mu) = 2\alpha_{sin}|\psi_{(b;\mu)}(\mathbf{r}, t)|^2 + 2\alpha_{dou}|\psi_{(b;\mu)}(\mathbf{r}, t)|^4 \quad (12)$$

The correction to the usual $|\psi|^2$ form of the single-particle detection distribution is in the form $|\psi|^4$.

We note the similarity between the last result and those obtained in quantum optics, where the probabilities of one- and two-photon absorption are, respectively, proportional to the first and second power of the quantum intensity [7]

The results obtained are independent of the mechanism of detection, ionization or any other, and are valid for any scheme of detection where various particles can lead to single detection events (Eqs. (1) and (2) are independent of the underlying detection mechanism).

5 Conclusions

We have analyzed in this Letter a new channel for the ionization of atoms and molecules by massive particles; the ionization by interaction with various particles, a massive counterpart to the well-known phenomenon of multi-photon absorption. In particular, this phenomenon would lead to *multiple detection events*. The results obtained here would also be valid for any detection process where the underlying physical mechanism leads to both types of detections, with one or various probe particles. The main conclusion is that these events would give rise to a new term, which when the two particles have the same mode distribution and spin is proportional to the fourth power of the modulus of the wavefunction. This distribution can, in principle, be distinguished from the usual one associated with single detection events (proportional to the second power of the modulus of the wave function): if $\alpha_{dou}/\alpha_{sin}$ is not too small both types of distributions could be distinguished experimentally. In particular, in atom lasers it seems to be a realistic goal. The most pictorial manifestation of this behaviour would be in interferometric arrangements [3, 8]. Placing the

detector at many different locations in a large number of repetitions of the experiment we could obtain the experimental curve of the interference experiment. Instead of the usual form (proportional to $|\psi|^2$) of the experiment with single-particle beams we would obtain one of the form $\alpha_{sin}|\psi|^2 + \alpha_{dou}|\psi|^4$, showing a small correction (of the order $\alpha_{dou}/\alpha_{sin}$) to the usual pattern.

We have only considered the case of two-particle beams. The generalization to beams with more particles is straightforward. The properties of the multi-particle detection can also depend on the type of incident state: in quantum optics it has been shown that the absorption efficiency depends on the statistical properties [9] and entanglement [10] of the incident light.

Most detectors are not sensitive to the spin state of the incident particles. The results corresponding to this particular situation can be obtained directly from our expressions. For incident identical particles the dependence of the two-particle detection on the modulus of the wavefunction is in the fourth power, just as in the example of the previous Section.

The results presented in this Letter show once more the notorious differences between bosons and fermions. The double sign in all the expressions for the probability distributions leads to additions for bosons and subtractions for fermions. The extreme situation occurs when both incident particles are in very similar states. Then the bosons tend towards maximum detection probabilities, whereas the fermion probabilities tend to small values (in the limit of equal states to null probabilities).

References

- [1] S. L. Rolston, W. D. Phillips, *Nature* 416 (2002) 219.
- [2] N. B. Delone, V. P. Krainov, *Multiphoton Processes in Atoms*, Springer, Heidelberg, 1994.
- [3] M. R. Andrews, C. G. Townsend, H. -J. Miesver, D. S. Durfee, D. M. Kurn, W. Ketterle, *Science* 275 (1997) 637.
- [4] Geiger counters and bubblechambers are devices too big to enter into this category of small size detectors. However, a simple trick can overcome this difficulty. We cover the device with a material impenetrable for the incident beam, except in a small size window, through which the particles can enter into the detector.
- [5] R. J. Glauber, *Quantum Optics and Electronics*, Edited by C. DeWitt, A. Blandin, C. Cohen-Tannoudji, Gordon and Breach, New York, 1965.
- [6] P. Sancho, *J. Phys. A* 37 (2004) 11003.
- [7] R Loudon, *The Quantum Theory of Light*, Clarendon Press, 1973.

- [8] A. Gammal, A. M. Kamchatnov, Phys. Lett. A 324 (2004) 227.
- [9] J. Kasiński, S. Chudzyński, W. Majewski, M. Glódź, Opt. Commun. 12 (1974) 304.
- [10] J. Peřina Jr, B. E. A. Saleh, M. C. Teich, Phys. Rev. A 57 (1998) 3972.